Computational Vision
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Lecture 21: Texture

Initialize

■ Spell check off, plot options, etc..

In[1]:=
```
Off[General::spell1];
```

In[2]:=
```
SetOptions[ArrayPlot, ColorFunction → "GrayTones", DataReversed → True,
  Frame → False, AspectRatio → Automatic, Mesh → False,
  PixelConstrained → True, ImageSize → Small];
SetOptions[ListPlot, ImageSize → Small];
SetOptions[Plot, ImageSize → Small];
SetOptions[DensityPlot, ImageSize → Small, ColorFunction → GrayLevel];
nbinfo = NotebookInformation[EvaluationNotebook[]];
dir =
 ("FileName" /. nbinfo /. FrontEnd`FileName[d_List, nam_, ___] ⧴
    ToFileName[d]);
```

■ Histogram

In[7]:=
```
histogram[image_, nbin_] := Module[{histx},
   Needs["Statistics`DataManipulation`"];
   histx = BinCounts[Flatten[image], {0, nbin - 1, 1}];
   Return[N[histx / Plus @@ histx]];
  ];
```

■ Entropy

In[8]:=
```
entropy[probdist_] := Plus @@ (If[# == 0, 0, -# Log[2, #]] & /@ probdist)
```

Outline

Last time

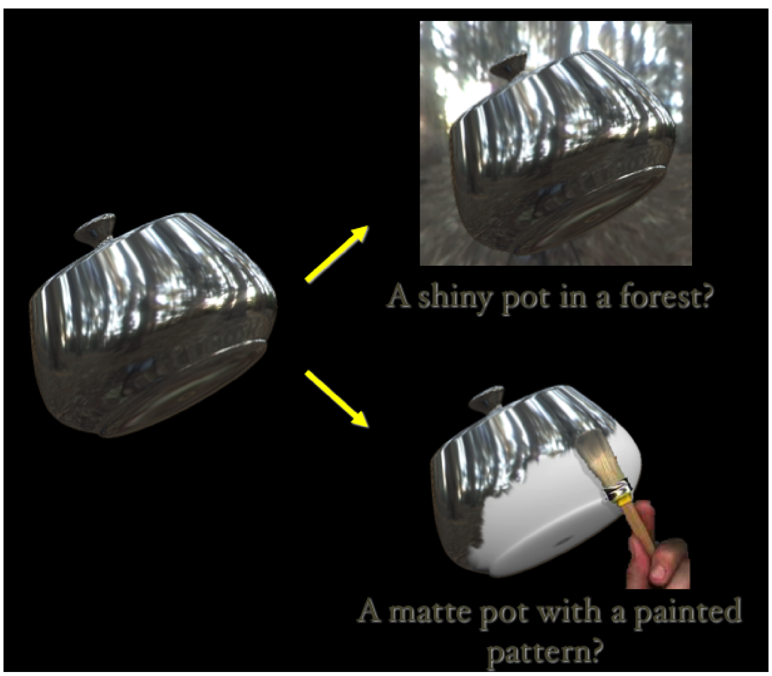
Surface material:

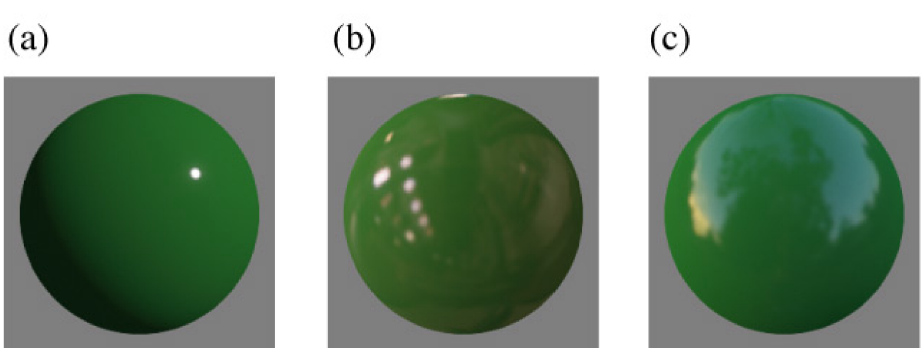
Surface properties, color, transparency, etc..

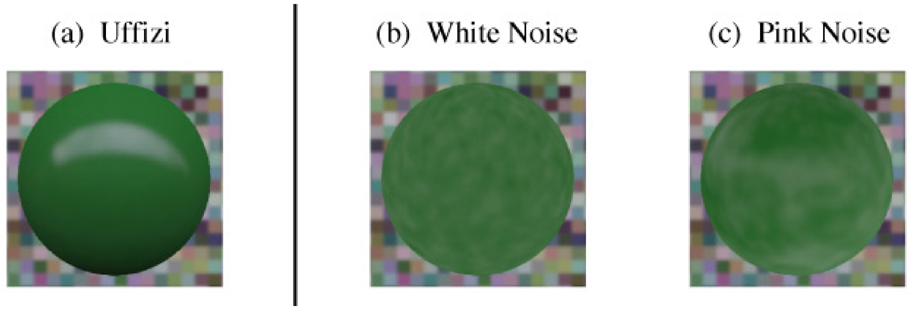
Reflectance & lightness constancy

Perception of shiny materials

Shiny or matte?







From: Fleming RW, Dror RO, Adelson EH (2003) Real-world illumination and the perception of surface reflectance properties. J Vis 3:347-368.

A major invariance problem.

Note in the above figure from Fleming et al. that the simple model of illumination with just one light source is not as effective as rendering in a realistic environment (Uffizi). But it isn't complexity per se, because white noise isn't good for conveying the underlying surface shininess.

One of the main conclusions is that the presence of edges and bright points important, rather than recognizable reflected objects.

http://journalofvision.org/3/5/3/article.aspx

For background on HDR illumination probe measurements, see: http://www.pauldebevec.com/Probes/

And on the Uffizi probe see too:

http : // commons.wikimedia.org / wiki / Image : HDR_example _ – _exposure.jpeg

■ Motion and shininess

http://gandalf.psych.umn.edu/~kersten/kersten-lab/demos/MatteOrShiny.html

Today

Generative models for texture classes

The "generic" natural image model

Is human vision "tuned" to natural image statistics?

Generative models for texture

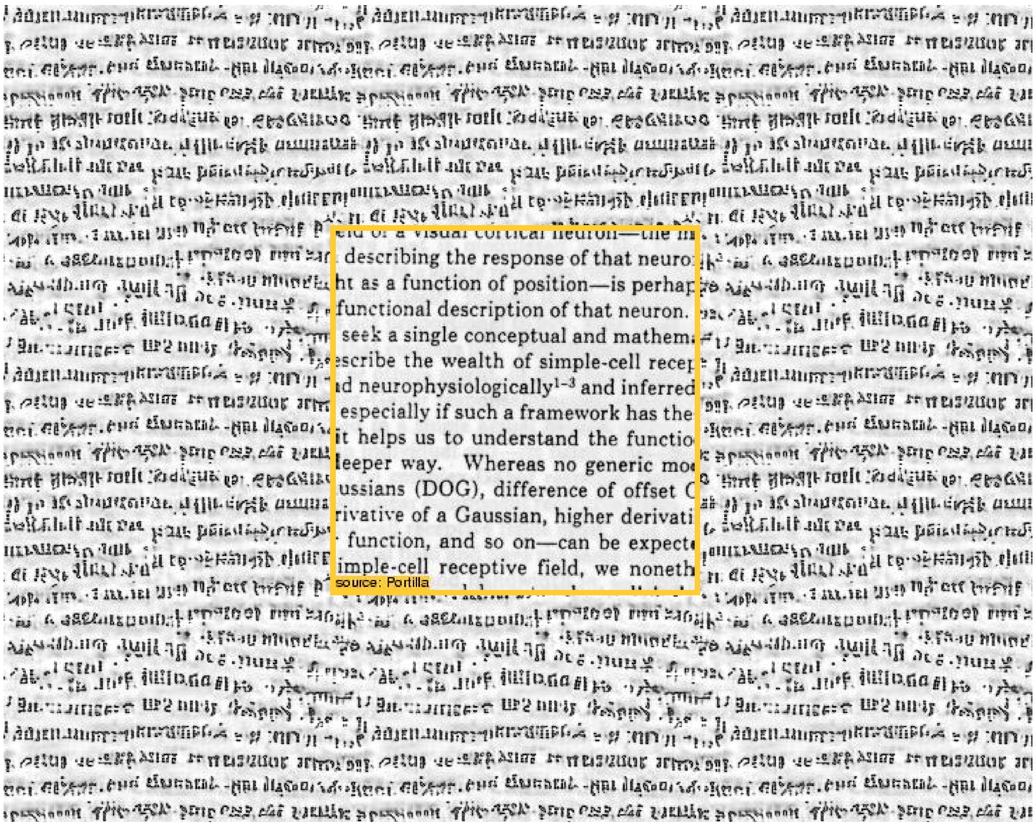
■ Databases

Types of textures. Deterministic, stochastic (Liu et al., 2010).

http://www.ux.uis.no/~tranden/brodatz.html http://sipi.usc.edu/database/

■ Text texture example from Javier Portilla and Eero Simoncelli

http://www.cns.nyu.edu/~eero/texture/



We'll focus on stochastic textures because of their close relationship to many textures typically encountered in nature.

Imagine an image ensemble consisting of all 256x256 images of "grass". This set is unimaginably large, yet there is a set of characteristic features that are common to all these images. Imagine we have an algorithm that from knowledge of these features generate random image samples from this imaginary ensemble. One kind of algorithm takes a white noise image as input, and produce as output image samples that resemble grass. The white noise input behaves like fair roll of a high-dimensional die.

We show several methods for generating textures.

And then we give an outline of one method for discovering the features from a small number of sample images.

There have been a number of studies that seek to extract the essential features of a texture class (such as "grass" or "fur" or "all natural images"...) and then use these to build a texture synthesizer that produces new samples from the same texture class. A generative model provides a test of the extent to which the model has capture the essential statistics or features. And as we show at the end of this notebook, a generative model can also be used to text theories of the kinds of information that human vision has about an image ensemble.

First-order intensity statistics. One of the simplest ways to do this would be to take what you've learned about intensity histograms, and then write a program that would produce new images by drawing pixel intensities from your model histogram, assuming each pixel is independent of the others. In other words, make i.i.d. random draws without consideration of any other pixel values.

Human vision shows sensitivity to first-order intensity statistics. For example, see Chubb et al., 2004. Motoyoshi et al. (2007) showed a correlation between skew symmetry and the perception of material gloss. But simple statistics can only go so far, Kim et al. (2011).

Make a random image generator that draws samples from an intensity histogram measured from an natural image

---

Random Fractals

Second-order intensity statistics. Recall that one way to characterize the second-order statistics of a natural image is in terms of its auto-correlation function.

Also recall that the Fourier transform of the autocorrelation function is equal to the spatial power spectrum of an image.

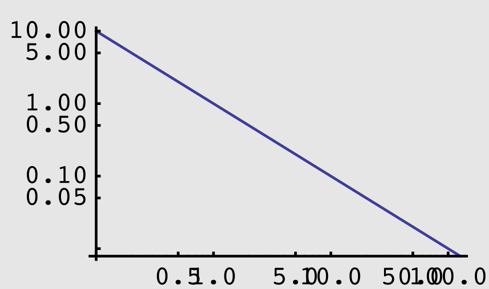
Natural images tend to have spatial frequency power spectra that fall off linearly with log spatial frequency (Simoncelli and Olshausen). When the slope of the fall-off is within a certain range, such images are called random fractals. The slope is related to the fractal dimension.

Random fractals can be characterized by the fractal dimension D (3<D<4) and amplitude spectrum, $1/(f_x^2 + f_y^2)$^(4-D). The amplitude spectrum is thus a straight line when plotted against frequency in log-log coordinates. The condition If[ ] is used to include a fudge term (1/(2)^(q)) to prevent blow up near zero in the Module[ ] routine below.

```
size = 256;
```

Random fractals have been suggested as good statistical models for the amplitude spectra natural images. Here is one way of generating them.

```
D1 = 3.5;
q = 4 - D1;
LogLogPlot[If[(i ≠ 0 || j ≠ 0), 1 / (i * i + 0 * 0) ^ (q), 1 / (2) ^ (q)],
  {i, .1, size / 2 - 1}, ImageSize → Small]
```



- **Here is a function to make a low-pass filter with fractal dimension D. (D, here should be between 3 and 4). Note that we first make the filter centered in the middle, and then adjust it so that it is symmetric with respect to the four corners.**

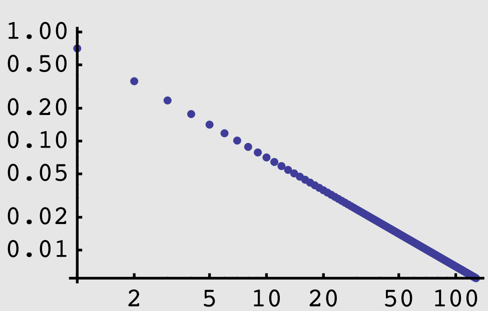
```
fractalfilter2[D_,size_] :=
Module[ {q,i,j,mat},
    q = 4 - D;
    mat = Table[If[(i != 0 || j!= 0),
        1.0/(i^2 + j^2)^q, 1.0/(2)^(q)],
    {i,-size/2,(size/2) - 1},{j,-size/2,(size/2) - 1}];
    Return[mat];
    ];
```

```
ft = Table[N[π (2 RandomReal[] - 1)], {i, 1, size}, {j, 1, size}];
ft = Fourier[ft]; randomphase = Arg[ft];
randomspectrum = Abs[ft];
```

```
fractalfilterarray = fractalfilter2[3.5, size];
ArrayPlot[fractalfilterarray^.2, Mesh → False]
```

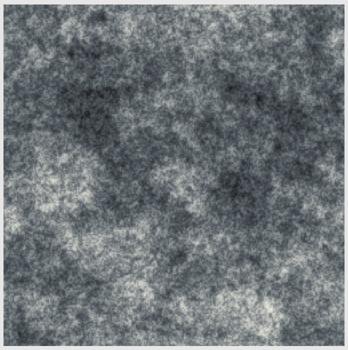
The exponent, .2, above is just for display purposes...it compresses the large values.

```
ListLogLogPlot[
 Table[RotateLeft[fractalfilterarray, {size / 2 + 1, size / 2 + 1}][[i, i]],
  {i, 1, size / 2}], ImageSize → Small]
```



- **Here is a random fractal image, with D = 3.5**

```
ArrayPlot[Chop[
InverseFourier[RotateLeft[fractalfilterarray,{size/2,size/2}] randomspectrum Exp[
Mesh->False]
```

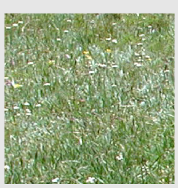


Texture synthesis using histogram matching

Histogram matching between two images.

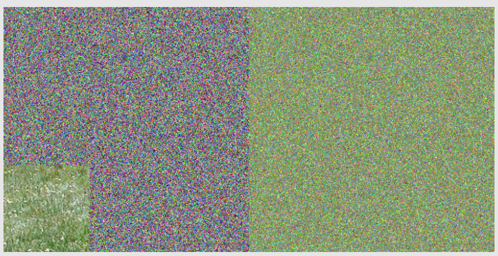
Let's go back to first-order intensity statistics. Suppose you have measured image statistics on a particular image, say a picture of grass. And now you want to force a white noise image to have the same statistics. Recall our earlier example of taking an natural image (a mountain lake) and forcing its intensity histogram to be flat ("whitening"). Suppose instead, we take a natural image, and want to force another image (perhaps a white noise image) to have the same histogram. This is called histogram matching. Histogram matching is useful when you want to have two images (or movies) look like they have the same lighting. But we can also ask whether we can use this method to make noise look more like texture from a specific texture class, like "grass".

Here's some example code from the *Mathematica* that does this for the three first-order color histograms.

```
img1 = 
```

```
;

img2 = RandomImage[NormalDistribution[.5, .2], {256, 256},
  ColorSpace → "RGB"];
```

```
𝒟1s = Map[HistogramDistribution[Flatten[#], 256] &,
   ImageData[img1, Interleaving → False]];
𝒟2s = Map[HistogramDistribution[Flatten[#], 256] &,
   ImageData[img2, Interleaving → False]];
{Tred, Tgreen, Tblue} =
  MapThread[
   FunctionInterpolation[InverseCDF[#1, CDF[#2, x]], {x, 0, 1},
     AccuracyGoal → 1] &, {𝒟1s, 𝒟2s}];
res = ImageApply[{Tred[#[[1]]], Tgreen[#[[2]]], Tblue[#[[3]]]} &, img2];
ImageAssemble[
 {ImageCompose[img2, ImageResize[img1, Scaled[.35]], {Left, Bottom},
   {Left, Bottom}], res}]
```



Statistics from image pyramids

The above sample on the right doesn't look much like grass, except for the color. To turn the noise sample into a texture that looks more like grass we need to force it to match additional statistics. Samples from the fractal process modeled above are multi-variate Gaussian. That would be one thing to try. But major limitation of Gaussian models is that they fail to capture phase structure, and in particular edges.

Heeger and Bergen (pdf) showed how to use image filter pyramids to generate novel textures from statistical "summaries" obtained from sample textures. They start off with a model of spatial filters that are selective for spatial frequency, orientation, and phase. The use of orientation filters captures oriented features of textures, and phase captures edges.

The filter model can be thought of as a model of the spatial filtering properties of V1 neurons. Then given a sample of a texture, measure the histograms for each of the filter outputs. The assumption is that these histograms summarize the essential features of the texture. Thus, given the histogram statistics, the goal of the algorithm is to produce new texture samples that have the same statistics but otherwise are random. One way to do this is to start of with a white noise sample (i.i.d. meaning each pixel is independently drawn from an identical distribution, such as a uniform or gaussian distribution), and then iteratively adjust the noise sample to have the same histograms as learned from the original natural texture sample. The general idea is like the above Mathematica example, except over a wider set of histograms.

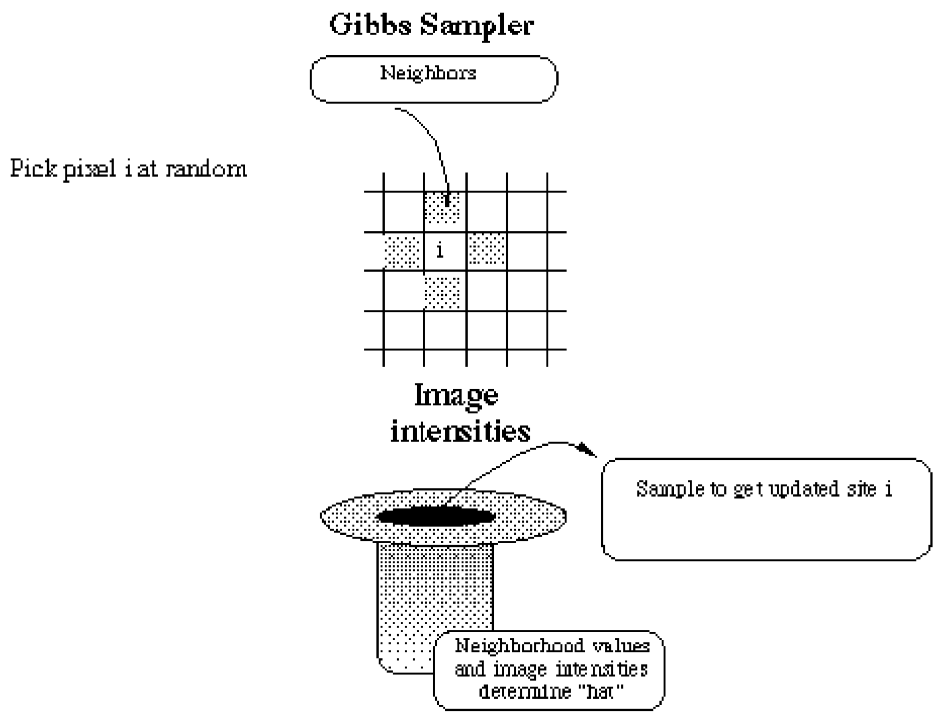
Texture synthesis using Markov Random Field models & Gibbs sampling

This next section shows another way to model textures using a model of the local conditional probabilities of intensities. In particular, we will show how to model piece-wise constant textures. The method it allows us, in theory, to generate samples from specified high-dimensonal joint probability functions. By clamping (or fixing) nodes with known measurements, versions of this method can also be used to do inference.

■ Modeling textures using Markov Random Fields

Markov Random Fields (MRFs) have been used in computer vision and graphics for many years, and there is substantial body of literature. Some of the earliest papers are from Cross, G. R., & Jain, A. K. (1983) and Geman and Geman (1984). Rather than go into the theory, we can get an intuitive sense for how MRFs can be used by looking at how they can be used to generate textures.

■ Sampling from textures using local updates



The Gibbs Sampler (Geman and Geman, 1984)

■ Set up image arrays and useful functions

In[88]:=

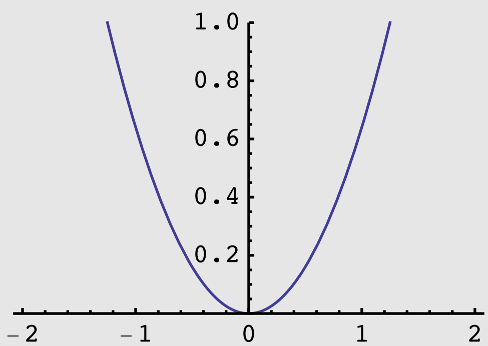
```
size = 32; T0 = 1.`; ngray = 16.`;
brown = N[Table[RandomReal[{1, ngray}], {i, 1, size}, {i, 1, size}]];
next[x_] := Mod[x, size] + 1;
previous[x_] := Mod[x - 2, size] + 1;
Plus @@ Flatten[brown]
---------------------- ;
Length[Flatten[brown]]
```

■ Gaussian potential

In[93]:=

```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := N[(x / s) ^2];
s0 = 1.25; n0 = 2;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

Out[96]=

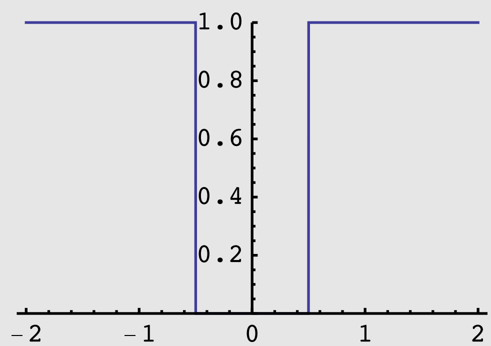


■ Ising potential

In[97]:=

```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := If[Abs[x] < .5, 0, 1];
(*f[x_,s_,n_]:=N[(x/s)^2];*)
s0 = 1.; n0 = 5;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

Out[100]=

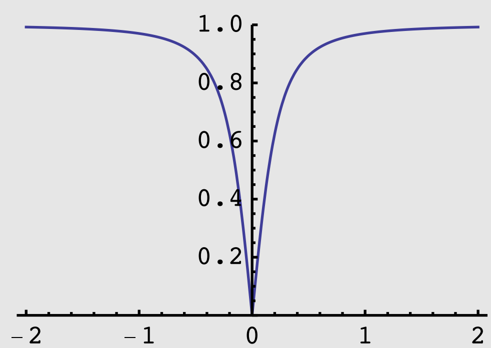


■ Geman & Geman potential

In[101]:=

```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := N[Sqrt[Abs[x / s]^n / (1 + Abs[x / s]^n)]];
(*f[x_,s_,n_]:=N[(x/s)^2];*)
s0 = .25; n0 = 2;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

Out[104]=



- **Define the potential function using nearest-neighbor pair-wise cliques**

In[105]:=

```
Clear[gibbspotential, gibbsdraw, tr];
gibbspotential[x_, avg_, T_] :=
  N[
   Exp[
    - (f[x - avg[[1]], s0, n0] + f[x - avg[[2]], s0, n0] +
        f[x - avg[[3]], s0, n0] + f[x - avg[[4]], s0, n0]) / T]];
```

- **Define a function to draw a single pixel gray-level sample from a conditional distribution determined by pixels in neighborhood**

In[107]:=

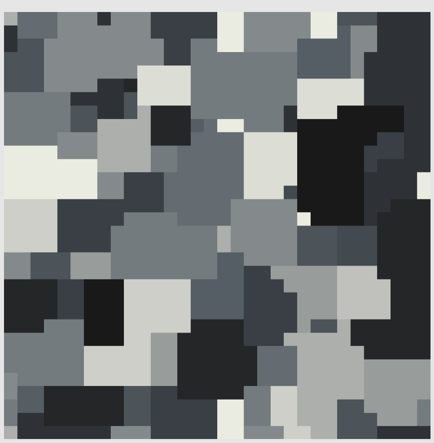
```
gibbsdraw[avg_, T_] :=
  Module[{}, temp = Table[gibbspotential[x + 1, avg, T], {x, 0, ngray - 1}];
   temp2 = FoldList[Plus, temp⟦1⟧, temp];
   temp10 = Table[{temp2⟦i⟧, i - 1}, {i, 1, Dimensions[temp2]⟦1⟧}];
   tr = Interpolation[temp10, InterpolationOrder → 0];
   maxtemp = Max[temp2]; mintemp = Min[temp2];
   ri = RandomReal[{mintemp, maxtemp}]; x = Floor[tr[ri]];
   Return[{x, temp2}];];
```

- **"Drawing" a texture sample**

In[108]:=

```
gd = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}];
Dynamic[gd]
```

Out[109]=



In[110]:=

```
For[iter = 1, iter ≤ 10, iter++, T = 0.25`;
 For[j1 = 1, j1 ≤ size size, j1++, {i, j} = RandomInteger[{1, size}, 2];
  avg = {brown⟦next[i], j⟧, brown⟦i, next[j]⟧, brown⟦i, previous[j]⟧,
    brown⟦previous[i], j⟧}; brown⟦i, j⟧ = gibbsdraw[avg, T]⟦1⟧;];
 gd = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}]]
```

Was it a true sample? Drawing true samples means that we have to allow sufficient iterations so that we end up with images whose frequency corresponds to the model. How long is long enough?

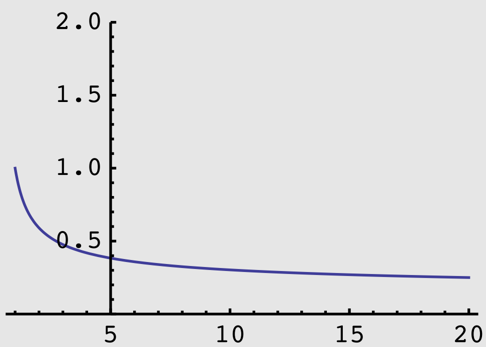
Finding modes

■ Define annealing schedule

In[111]:=

```
anneal[iter_, T0_, a_] := T0 * (1 / a) / (1 / a + Log[iter]);
Plot[anneal[iter, T0, 1], {iter, 1, 20}, PlotRange → {0, 2}]
```

Out[112]=



■ "Drawing" a texture sample with annealing

In[113]:=

```
gd2 = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}];
Dynamic[gd2]
```

Out[114]=

In[115]:=
```
For[iter = 1, iter ≤ 10, iter++, T = anneal[iter, T0, 1];
  For[j1 = 1, j1 ≤ size size, j1++, {i, j} = RandomInteger[{1, size}, 2];
   avg = {brown⟦next[i], j⟧, brown⟦i, next[j]⟧, brown⟦i, previous[j]⟧,
     brown⟦previous[i], j⟧}; brown⟦i, j⟧ = gibbsdraw[avg, T]⟦1⟧;];
  gd2 = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}]];
```

Machine learning of distributions on textures

A fundamental problem in learning image statistics that are sufficient for generalization and random synthesis is that images have enormously high dimensionality compared with the size of a reasonable database. The is one of the problems of the “curse of dimensionality”. One method to deal with this is to seek out probability distributions that have the same statistics (i.e. a small finite set of statistical features) as those measured from an available database (e.g. "1000 pictures of grass"), but are minimally constraining in other dimensions. Suppose one has a collection of probability distributions that all have the same statistics. At one extreme, the original database itself defines a distribution--a random draw is just a pick of one of the pictures. But this distribution has no "creativity" and leaves out a huge set of grass images that are not in the database. At the other extreme, is the maximum entropy distribution (Cover and Thomas, 1991).

Minimax entropy learning: Zhu et al.

This section provides a brief outline of work by Zhu, S. C., Wu, Y., & Mumford, D. (1997). Minimax Entropy Principle and Its Applications to Texture Modeling. Neural Computation, 9(8), 1627-1660. It provides a theoretical framework for learning “flat” models of images. Recent work seeks to extend this theory to hierarchical models of image structure (Yuille, 2011).

See the References for other work on texture learning and modeling.

■ Maximum entropy to determine p_M(I) which matches the measured statistics, but is “least committal”

Suppose we have a set of filters ϕ_i. An example would be a simple difference filter such as a discrete approximation to a ∇^2operator, which we saw produces histograms from natural images with high kurtosis.

$$\{\phi_i (I) : i = 1, ..., N\} \tag{1}$$

Given a collection of image samples I, measure the average values of the filter outputs, i.e. texture statistics, ψ_i. These statistics could be the histogram values themselves, i.e. for each filter we would get n histogram probability estimates, where n is the number of bins.

A good model of the texture p_M would have the same statistics as the true underlying model model, p(I). (We don’t know p(I), but we have samples from it--the ones we used to calculate the statistics.)

$$\sum_I p_M (I)\, \phi_i (I) = \psi_i, \text{ for } i = 1, ..., N \tag{2}$$

But there is an enormous family of possible probability distributions $\{p_M(I)\}_M$ that could all have the same statistics.

If we want a texture model that has maximum “creativity”, we can model this constraint by looking for the distribution that has the highest entropy, but constrained to have the required statistics.

Zhu et al.'s method built on a a standard method in information theory (Cover and Thomas, 1991) to obtain the maximum entropy distribution for a given set of measured statistics. The idea was to "learn" the form of the potentials λ_i (as in the Ising potential assumed above).

$$p_M(I) = \frac{1}{Z[\lambda]} \exp\left\{-\sum_{i=1}^{N} \lambda_i \phi_i(I)\right\} \tag{3}$$

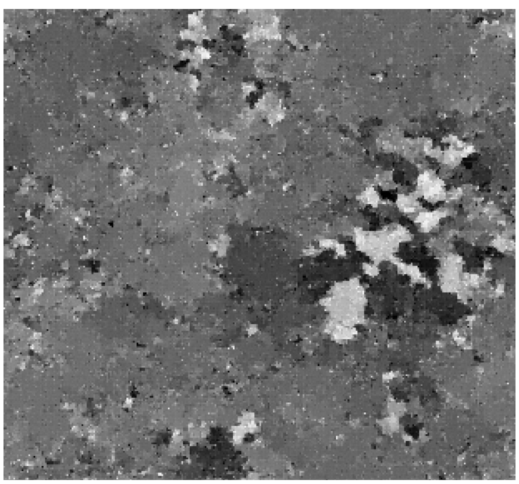
■ Minimum entropy to determine statistics/features

But what features (i.e. filters) are the most important? It will depend on the texture and the initial choice of feature set. Suppose one has a filter set modeled after V1 spatial filters. Some filters may be much more important than others in capturing the essential statistics. Assume that p(I) is the true model that has all of the essential statistics. This could be really complex, and we don't know for sure what filters to include. So Zhu et al.'s idea was to do something analogous to a Taylor series expansion, and order filters so that as one added more filters to p_M, it gets us closer to the true distribution $p(I)$. To do this, one needs a measure of "distance" between two distributions. We've already learned about d' in a completely different context. A more general measure is Kullbach-Leibler divergence (wiki): D(p(I) | p_M(I)). Zhu et al. showed that choosing filters that minimize the entropy of p_M(I), they could move the distribution in the direction towards p(I).

$$\sum_I p(I) \log p_M(I) = \sum_I p_M(I) \log p_M(I) \tag{4}$$

$$D(p(I) \mid p_M(I)) = \text{entropy}(p_M(I)) - \text{entropy}(p(I)) \tag{5}$$

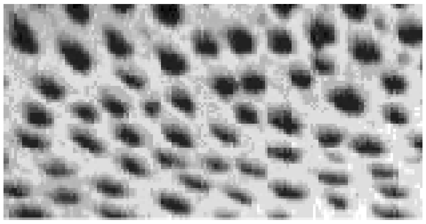
■ Sample from generic prior



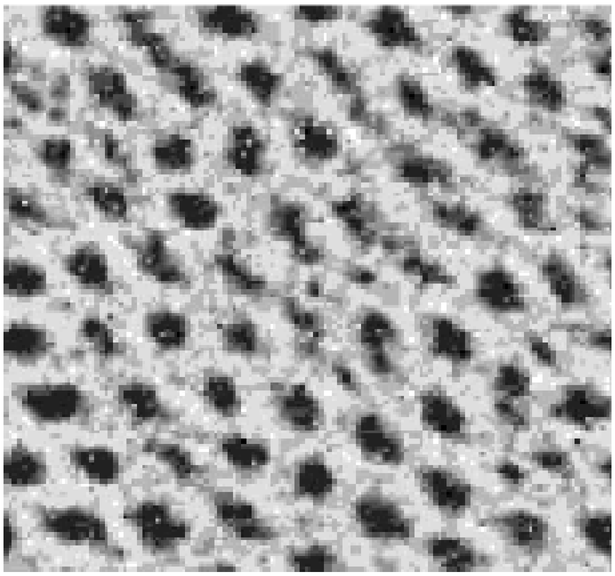
■ Sample from class-specific prior

Song Chun Zhu, Zhu & Mumford, IEEE PAMI, Zhu, Wu, Mumford, 1997

Original texture



Synthesized sample using Gibbs sampler



Nonparametric sampling

While theoretically well-grounded, the above approach can be difficult to implement efficiently. A more practical approach is suggested by Claude Shannon's approach to synthesizing English (Shannon, 1948; 1951).

Efros' application to textures. Instead of first estimating the local MRF distributions (conditional value of a pixel given its neighbors), one can imagine starting off with a small seed, and then querying the original sample image to find similar neighborhoods to constrain how to make the draws. See :

http://graphics.cs.cmu.edu/people/efros/research/EfrosLeung.html

References

Besag, J. (1972). Spatial interaction and the statistical analysis of lattice systems. {\it Journal of the Royal Statistical Society B}, {\bf 34}, 75-83.

Chubb, C., Landy, M. S., & Econopouly, J. (2004). A visual mechanism tuned to black. Vision Research, 44(27), 3223–3232. doi:10.1016/j.visres.2004.07.019

Clark, James J. \& Yuille, Alan L. (1990) Data fusion for sensory information processing systems. Kluwer Academic Press, Norwell, Massachusetts.

Cross, G. C., & Jain, A. K. (1983). Markov Random Field Texture Models. IEEE Trans. Pattern Anal. Mach. Intel., 5, 25-39.

Cover TM, Thomas J, A. (1991) Elements of Information Theory. New York: John Wiley & Sons, Inc.

Geiger, D., & Girosi. (1991). Parallel and Deterministic Algorithms from MRF's: Surface Reconstruction. I.E.E.E PAMI, 13(5).

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